Parallel Continuum Robots

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Introduction

The work presents an investigation into the kinematics and simulation of a 6-$\overline{R}US$ parallel kinematic mechanism (PKM). The study commences with the formulation of inverse kinematics using geometric method, successfully deriving joint angles for various poses for a rigid PKM. Subsequently, the model is succesfully implemented in the PyBullet physics engine via URDF representation, accompanied by the trajectory comparison with the geometry-based method. The research also delves into the realm of parallel continuum robots (PCR). The static inverse kinematics problems for such manipulators is formulated as the solution to multiple Cosserat-rod models with coupled boundary conditions [1].

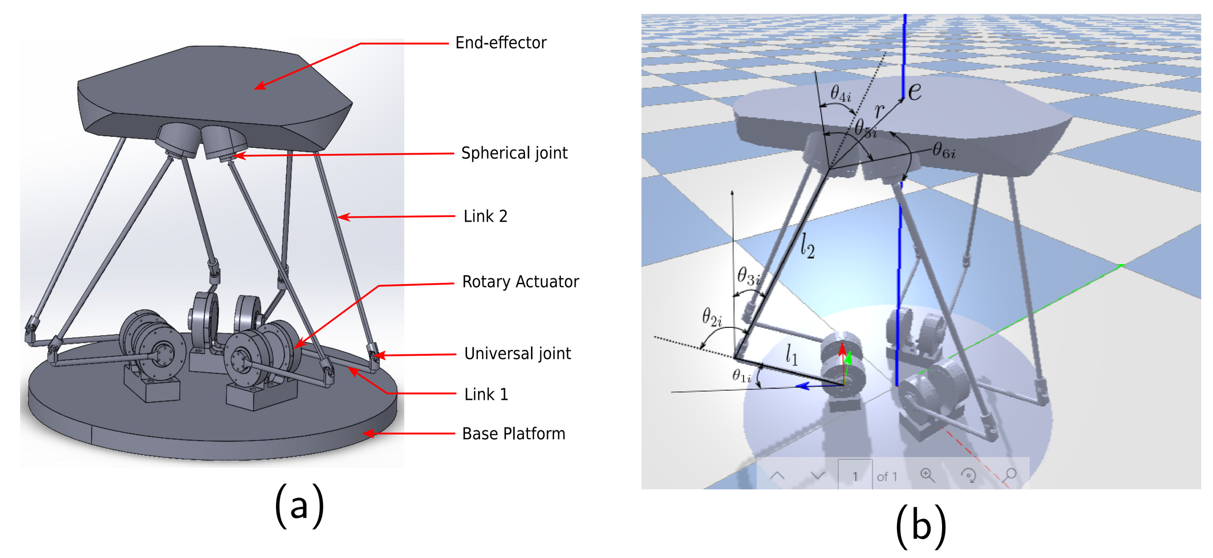


Figure: (a) The hardware setup of the proposed 6-DOF 6-$\overline{R}US$ rigid parallel kinematic manipulator. (b) Kinematic definition for the rigid PKM

Analytical Inverse Kinematic (AIK) formulation

The PKM is a 6-DOF 6-$\overline{R}US$ rigid Stewart-Gough platform (RSGP) where each leg can be manually actuated using a rotary motor as shown in the figure (a) above. The inverse kinematics problem involves finding the values of the six revolute joint angles $(q1, \dots, q6)$ that result in a desired end-effector (EE) position (x, y, z) and orientation $(roll, pitch, yaw)$. This problem is solved using geometric method which involves using trigonometry and geometry to derive closed-form solutions for the joint angle [2].

$$\theta_{3i} = arcsin(\frac{y}{l_{2i}}) \tag{1}$$

$$cos(\theta_{2i}) = \frac{x^2 + z^2 - l_{1i}^2 - (l_{2i}cos(\theta_{3i}))^2}{2l_{1i}l_{2i}cos(\theta_{3i})cos(\theta_{2i})};$$

$$\sin(\theta_{2i}) = \pm\sqrt{1 - \cos(\theta_{2i})^2}$$

$$\theta_{2i} = arctan2(\sin(\theta_{2i}), \cos(\theta_{2i})) \tag{2}$$

$$arctan2(\frac{z}{x}) - arctan2(\frac{l_{2i}cos(\theta_{3i})sin(\theta_{2i})}{l_{1i}^2 + l_{2i}cos(\theta_{3i})cos(\theta_{2i})}) \tag{3}$$

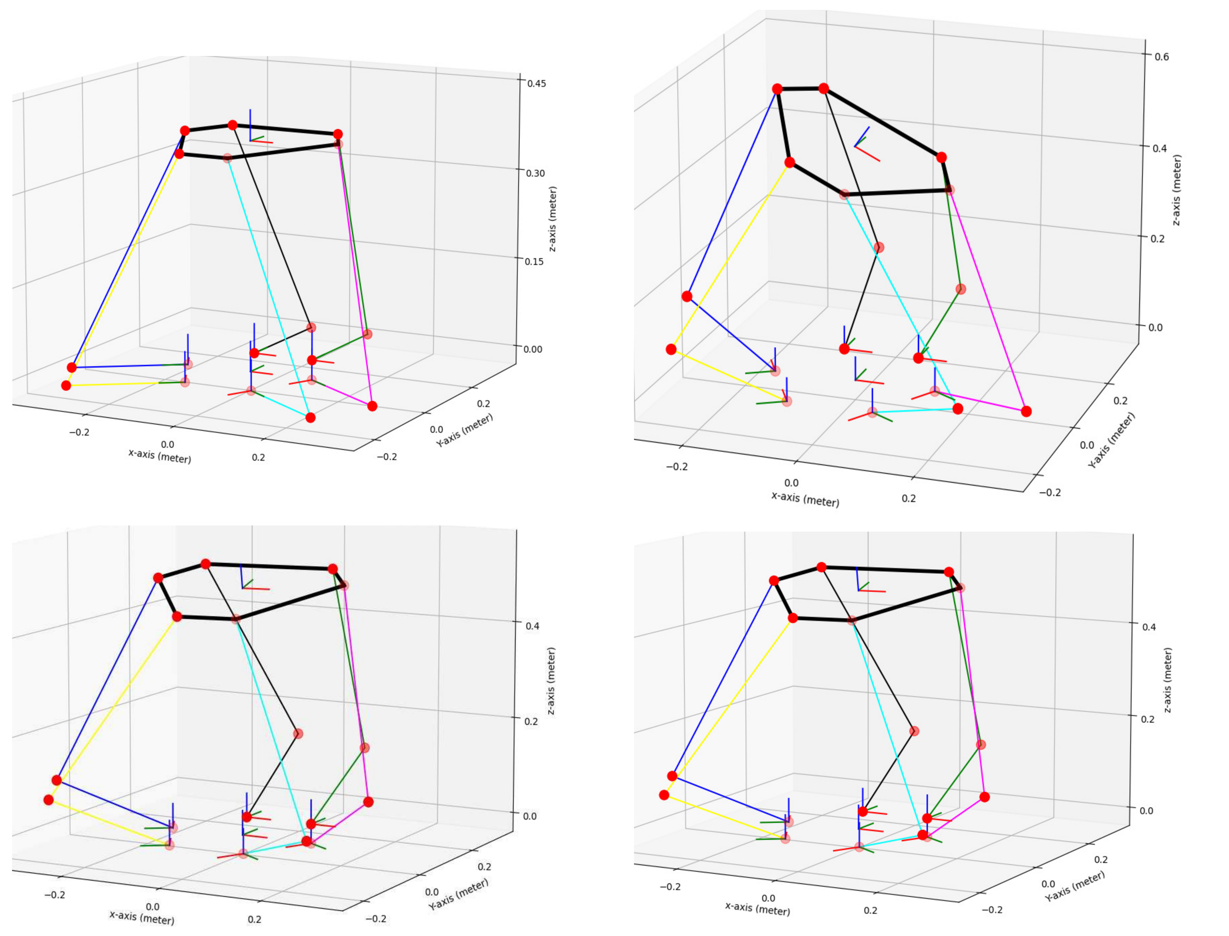


Figure: Ananlytical inverse kinematics solution for 6-$\overline{R}US$ PKM.

Pybullet Simulation for forward kinematics

The joint angles calculated from AIK were then used for the forward kinematic solution in Pybullet. Closed kinematic loops were achieved by using constraints library in the software.

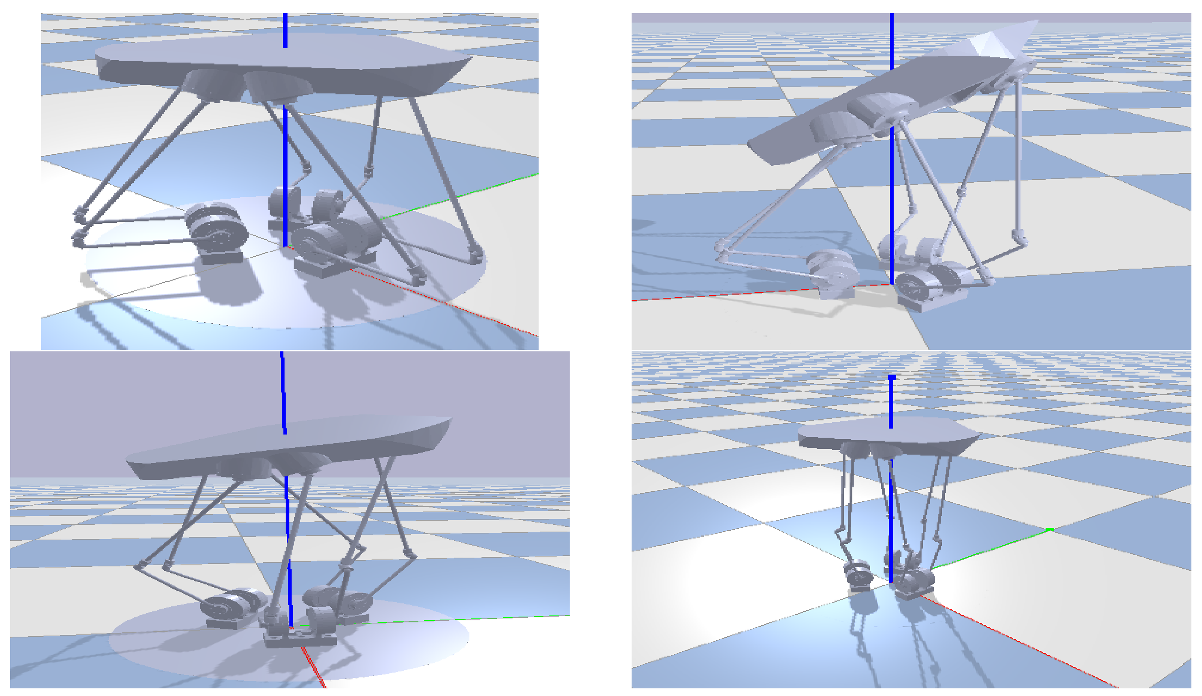


Figure: Ananlytical inverse kinematics solution for 6-RUS PKM.

Trajectory comparison between Pybullet and AIK

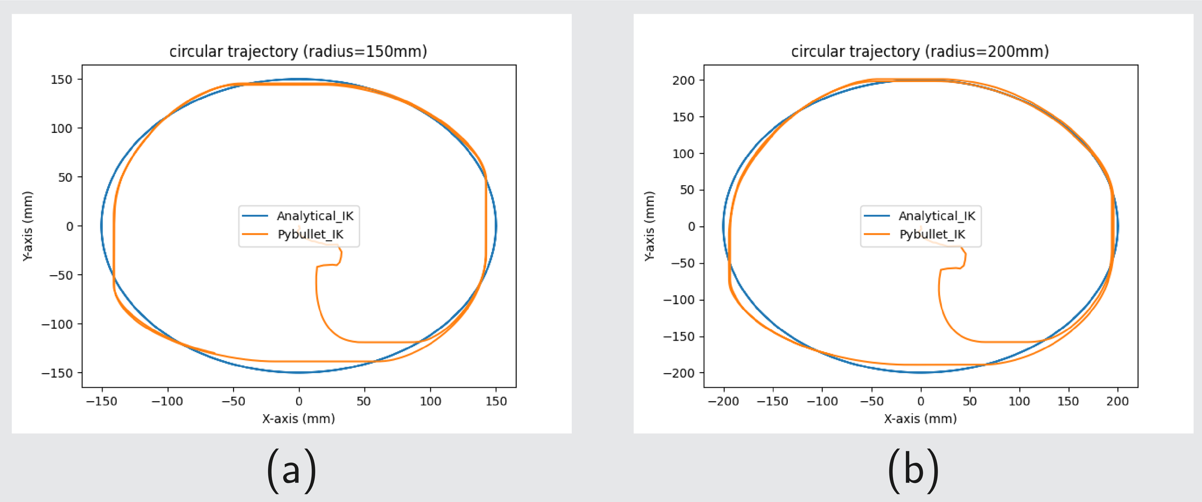


Figure: Circular trajectory comparison for (a) 0.15 radius and (b) 0.2 radius with the forward kinematic simulation using pybullet from the joint angles generated by AIK for the same circular trajectory

Parallel Continuum Manipulator (PACOMA)

PCR has high payload capacity, and acccuracy. It can be easily miniaturized and have low mass due to off-loading of the actuators It has inherent mechanical compliance which leverages manipulation in highly confined places. Applications: Medical Robotics, Space Exploration,Rehabilitation,etc

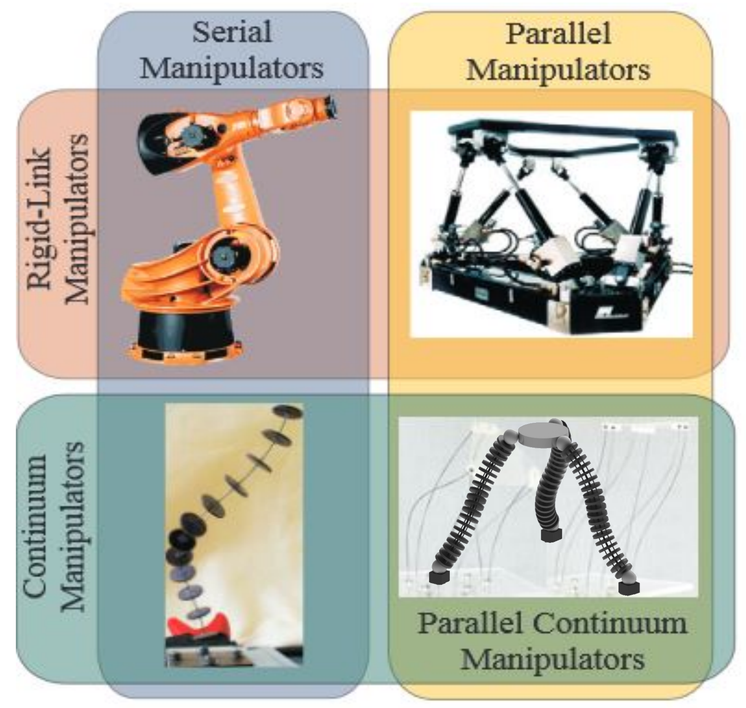


Figure: Our work aims to explore the relatively new domain of PCR situated in the 4th quadrant [1]

Mechanical design

- Six compliant legs
- Similar arrangement to RSGP
- Each leg can be manually actuated
- Titanium alloy material

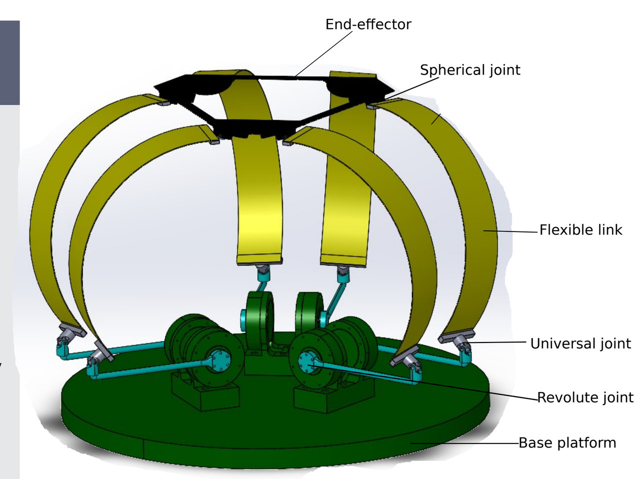
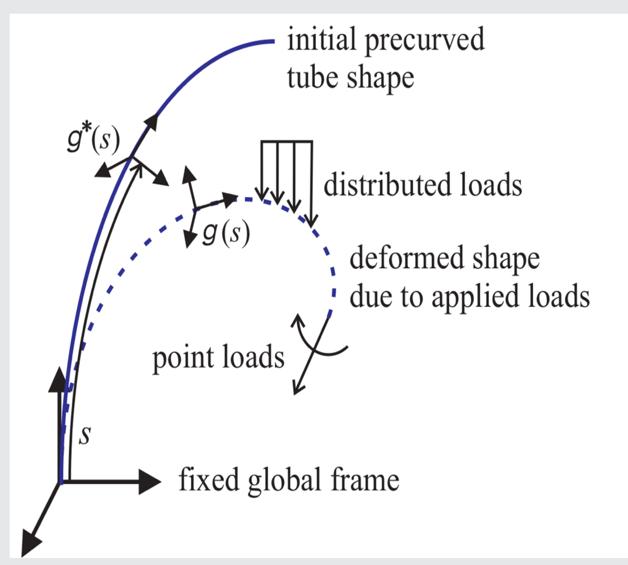


Figure: CAD model of PACOMA

Differential equations & Static equilibrium constraints [3]



$$\mathbf{p}_i' = \mathbf{R}_i\mathbf{v}_i; \quad \mathbf{p}_i \in \mathbb{R}^3$$
$$\mathbf{R}_i' = \mathbf{R}_i\hat{\mathbf{u}}_i; \quad \mathbf{R}_i \in SO(3)$$
$$\mathbf{n}_i' = -\mathbf{f}_i; \quad \mathbf{n}_i, \mathbf{m}_i \in \mathbb{R}^3$$
$$\mathbf{m}_i' = -\mathbf{p}_i' \times \mathbf{n}_i - \mathbf{l}_i$$

$$\mathbf{n}_i(s) = \mathbf{R}_i(s)\mathbf{K}_{SE,i}(\mathbf{v}_i(s) - \mathbf{v}_i^*(s))$$
$$\mathbf{m}_i(s) = \mathbf{R}_i(s)\mathbf{K}_{BT,i}(\mathbf{u}_i(s) - \mathbf{u}_i^*(s))$$

$$\mathbf{g}_i^*(s_i) = \begin{bmatrix} \mathbf{R}_i^*(s_i) & \mathbf{p}_i^*(s_i) \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{K}_{BT,i}(s) = \begin{bmatrix} EI_{xx,i}(s) & 0 & 0 \\ 0 & EI_{yy,i}(s) & 0 \\ 0 & 0 & G(I_{xx,i} + I_{yy,i}) \end{bmatrix}$$

$$\mathbf{K}_{SE,i}(s) = \begin{bmatrix} G_iA_i(s) & 0 & 0 \\ 0 & G_iA_i(s) & 0 \\ 0 & 0 & E_iA_i(s) \end{bmatrix}$$

Static Equilibrium equations

$$\sum_{i=0}^{n}[\mathbf{n}_i(l_i)] - \mathbf{F} = 0$$

$$\sum_{i=0}^{n}[\mathbf{p}_i(l_i) \times \mathbf{n}_i(l_i) + \mathbf{m}_i(l_i)] - \mathbf{p}_e \times \mathbf{F} - \mathbf{M} = 0$$

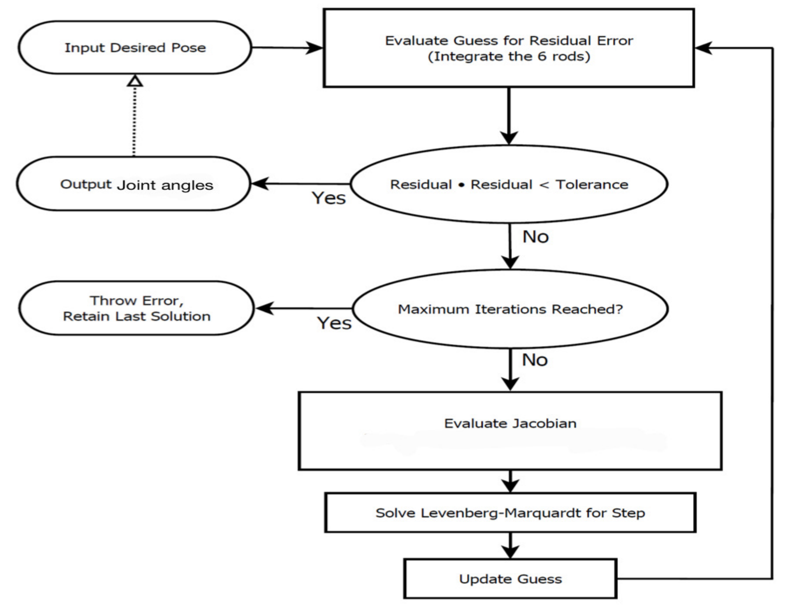


Figure: Higher level working of the solver [4]

Inverse Kinetostatic analysis for PACOMA

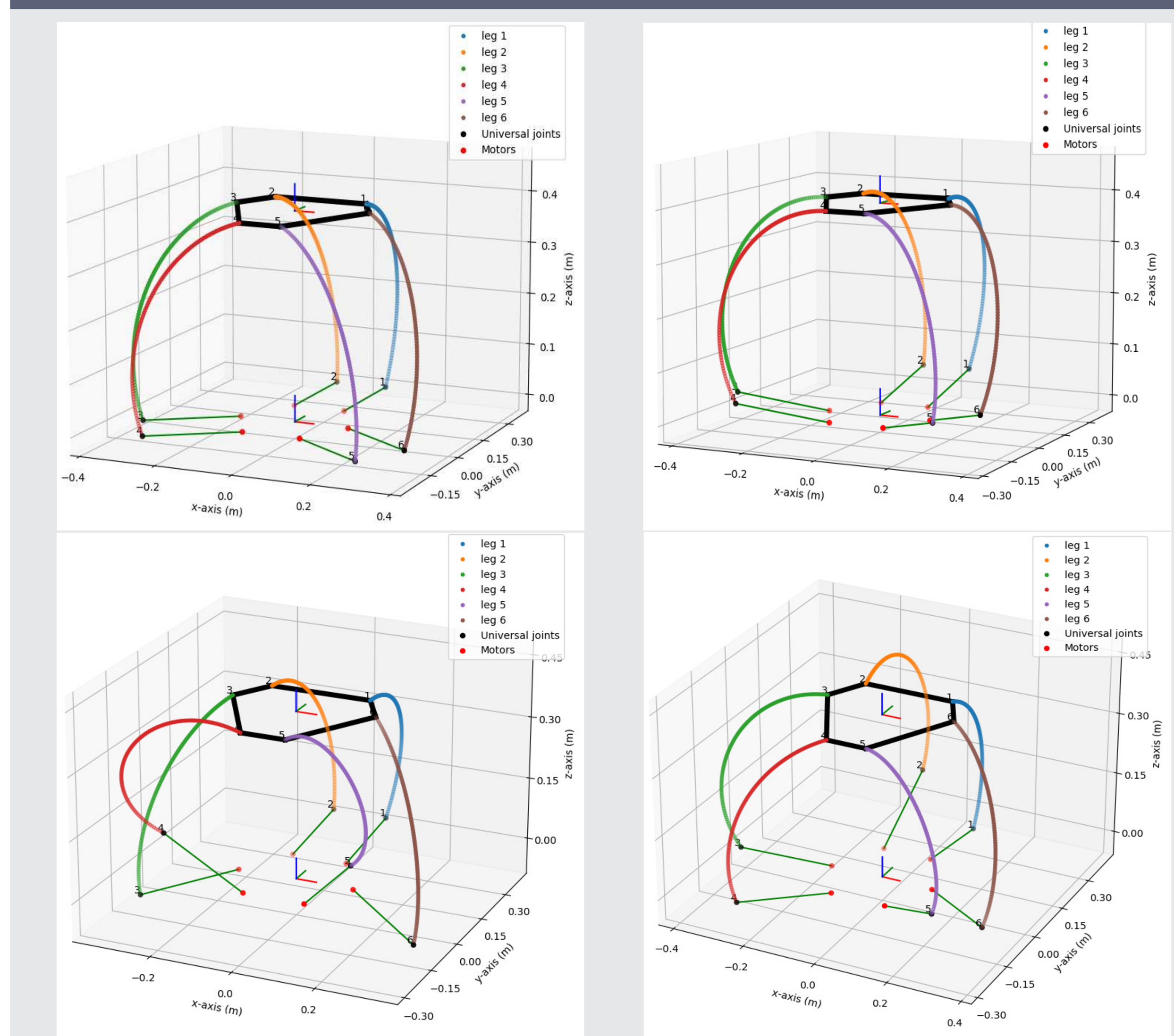


Figure: Joint angles are computed as a result of Kinetostatic force balance at EE

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References

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