Mixed Integer MPC for a Free-Floating Satellite Testbed

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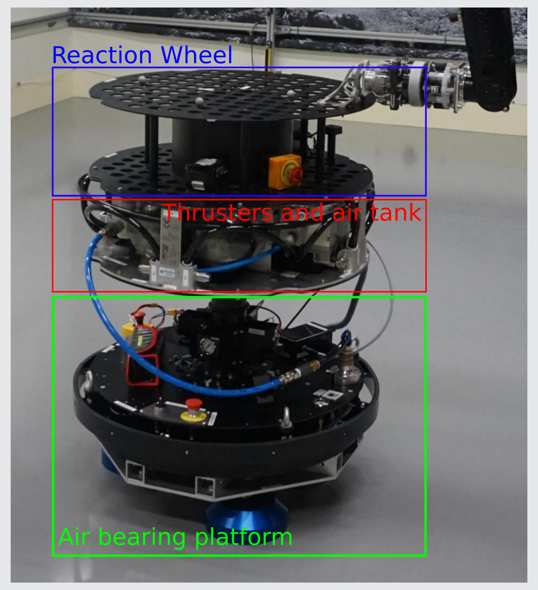
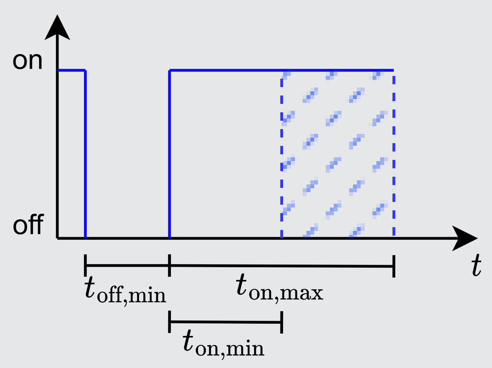
ESA's free-floating platform REACSA [1]

- 220 kg air-bearing platform
- floats on 9m × 5m flat-floor
- Reaction Wheel for precise torque control
  - RW speed limits lead to saturation
- 8 thrusters apply linear and angular acceleration
  - on/off (**binary** actuated) **thrusters**
  - thrusters have activation **time constraints**
    - minimum off time: $t_{\text{off,min}}$
    - minimum on time: $t_{\text{on,min}}$
    - maximum on time: $t_{\text{on,max}}$





Enforcing binary inputs ($\mathbf{u}_{\text{bin}} \in \mathbb{U}_{\text{bin}}$)

- Mixed Integer Program (MIP)

$$u_{i,t+k|t} \in \{0,1\}, \forall u_i \in \mathbf{u}_{\text{bin}}, \forall k \in [0,N) \quad (1)$$

Requires special solver

- Penalty Term

$$\sum_{j=1}^{3} 4\beta \left(u_{i,t+k|t} - u_{i,t+k|t}^2\right),\ \beta > 0,\ \forall u_i \in \mathbf{u}_{\text{bin}} \quad (2)$$

Becomes non-convex Quadratic Program (QP)

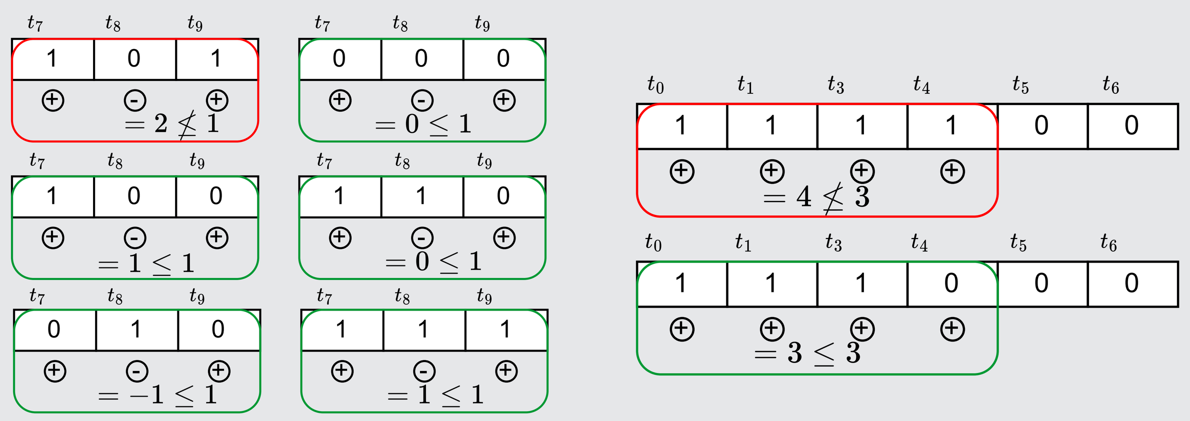
- Linear Complementarity Constraints (LCC)

$$0 \le (1 - u_{i,t+k|t}) \perp u_{i,t+k|t} \ge 0,\ \forall u_i \in \mathbf{u}_{\text{bin}} \quad (3)$$

Becomes Mathematical Program with Complementarity Constraints (MPCC)

Activation time constraints ($\mathbf{u}_{\text{bin}} \in \mathbb{U}_{\text{time}}$)

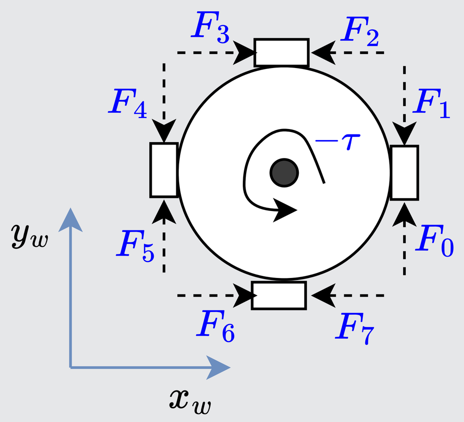
- minimum off time: $t_{\text{off,min}} = 0.1\,\text{s} = \Delta t$
  Enforced naturally by zero-oder hold
- minimum on time: $t_{\text{on,min}} = 0.2\,\text{s} = 2\Delta t$
  $+u_{i,t+k-1|t} - u_{i,t+k|t} + u_{i,t+k+1|t} \le 1,\ \forall k \in [-2, N-1), \forall i \in \mathbf{u}_{\text{bin}}$
- maximum on time: $t_{\text{on,max}} = 0.3\,\text{s} = 3\Delta t$
  $\sum_{j=k}^{k+3} u_{i,t+j|t} \le 3, \forall k \in [-3, N-3), \forall i \in \mathbf{u}_{\text{bin}}$



System model

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{0}^{3\times4} & \mathbf{I}^{3\times3} \\ \mathbf{0}^{4\times7} & \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x & y & \theta & \dot{x} & \dot{y} & \dot{\theta} & \omega_{\text{RW}} \end{bmatrix}^T}_{\mathbf{x}} \quad (4)$$

$$+ \underbrace{\begin{bmatrix} & & & & \mathbf{0}^{3\times9} & & & & \\ 0 & -s\theta\frac{F_n}{m} & s\theta\frac{F_n}{m} & -c\theta\frac{F_n}{m} & c\theta\frac{F_n}{m} & s\theta\frac{F_n}{m} & -s\theta\frac{F_n}{m} & c\theta\frac{F_n}{m} & -c\theta\frac{F_n}{m} \\ 0 & c\theta\frac{F_n}{m} & -c\theta\frac{F_n}{m} & -s\theta\frac{F_n}{m} & s\theta\frac{F_n}{m} & -c\theta\frac{F_n}{m} & c\theta\frac{F_n}{m} & s\theta\frac{F_n}{m} & -s\theta\frac{F_n}{m} \\ \frac{-1}{I_S} & \frac{F_n r}{I_S} & -\frac{F_n r}{I_S} & \frac{F_n r}{I_S} & -\frac{F_n r}{I_S} & \frac{F_n r}{I_S} & -\frac{F_n r}{I_S} & \frac{F_n r}{I_S} & -\frac{F_n r}{I_S} \\ \frac{1}{I_{\text{RW}}} & & & & \mathbf{0}^{1\times8} & & & & \end{bmatrix}}_{\mathbf{B}(\theta)} \underbrace{\begin{bmatrix} \tau \\ \mathbf{u}_{\text{bin}} \end{bmatrix}}_{\mathbf{u}} \quad (5)$$



With $\tau \in \mathbb{R}$ and $\mathbf{u}_{\text{bin}} = \begin{bmatrix} u_0 & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \end{bmatrix}^T \in \{0,1\}^8$

Model Predictive Control formulation

Finite horizon optimal control problem:

$$J^*(\mathbf{x}_t) = \min_{\mathbf{U}_t, \mathbf{X}_t} \mathcal{L}_f(\mathbf{x}_{t+N|t}) + \sum_{k=0}^{N-1} \mathcal{L}(\mathbf{x}_{t+k|t}, \mathbf{u}_{t+k|t}) \quad (6a)$$

$$\text{s.t. } \mathbf{x}_{t+k+1|t} = \mathbf{x}_{t+k|t} + \Delta t \mathbf{A} \mathbf{x}_{t+k+1|t} + \Delta t \mathbf{B} \mathbf{u}_{t+k|t}, \quad \forall k \in [0, N) \quad (6b)$$

$$-\tau_{\max} \le \mathbf{u}_{0,t+k|t} \le \tau_{\max}, \quad \forall k \in [0, N) \quad (6c)$$

$$\mathbf{x}_{\text{lb}} \le \mathbf{x}_{t+k|t} \le \mathbf{x}_{\text{ub}}, \quad \forall k \in [0, N) \quad (6d)$$

$$\mathbf{x}_{\text{f,lb}} \le \mathbf{x}_{t+N|t} \le \mathbf{x}_{\text{f,ub}} \quad (6e)$$

$$\mathbf{u}_{\text{bin},t+k|t} \in \mathbb{U}_{\text{bin}}, \quad \forall k \in [0, N) \quad (6f)$$

$$\mathbf{u}_{\text{bin},t+k|t} \in \mathbb{U}_{\text{time}}, \quad \forall k \in [0, N) \quad (6g)$$

$$\mathbf{x}_{t|t} = \mathbf{x}_t \quad (6h)$$

- Discretization $\Delta t = 0.1\,\text{s}$
- Closed loop control cycle with 100 ms

Feasibility analysis

On a simplified model (4 thrusters, no reaction wheel) the feasibility of all three binary input formulations is compared:

Linear Mixed Integer:

- Feasible solutions within 100 s
- For short prediction horizons solutions are optimal enough

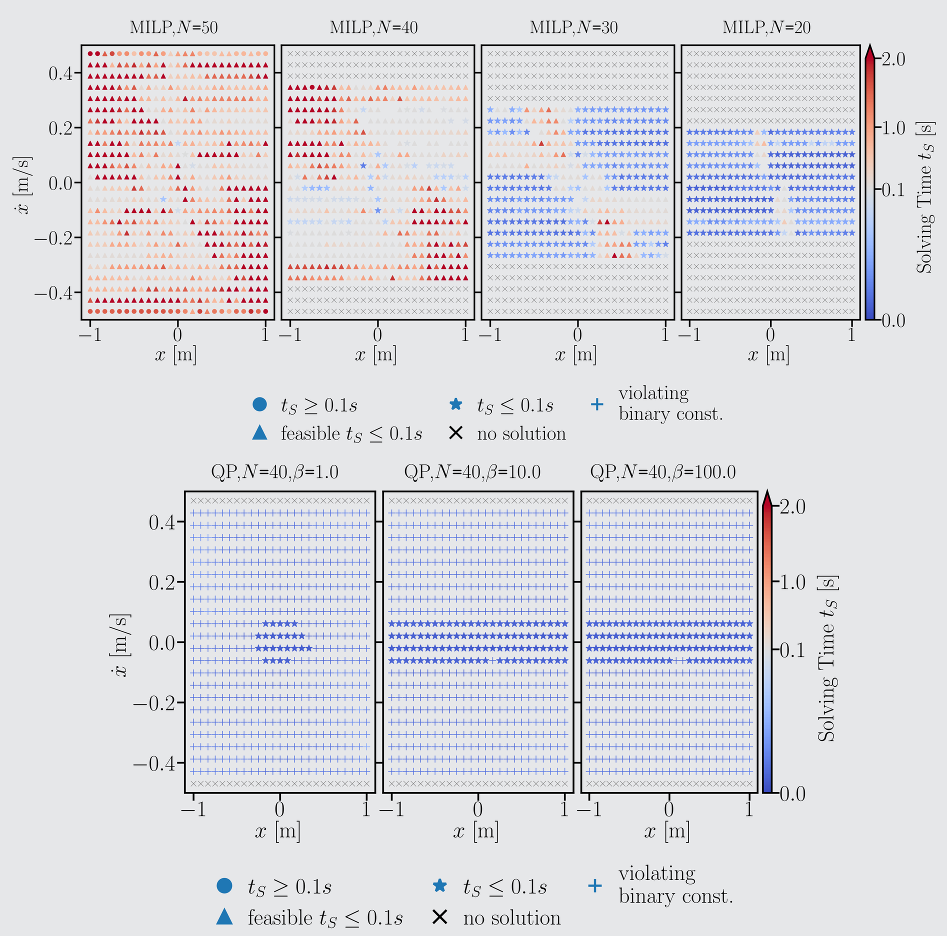
(Quadratic) Penalty-term:

- Penalty term not fully minimized
- Solutions have continuous values

Complementarity constraints:

- For this problem most of the time infeasible

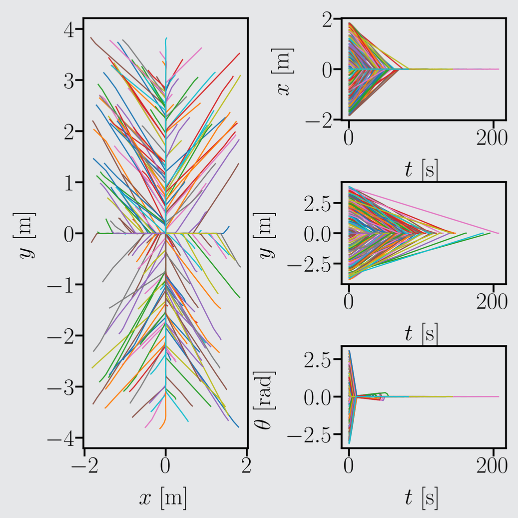
$\Longrightarrow$ **Mixed Integer Linear MPC is used in this work**



Simulation results

MILP was tested on 200 experiments with random initial states. Simulated using *drake toolbox*[2].

- Average solver time 56.08 ms with a standard deviation of 27.00 ms
- 27.45 % suboptimal solution
- In all experiments the system was steered towards and kept at the origin with an RMS error of 0.004 m and 0.097°



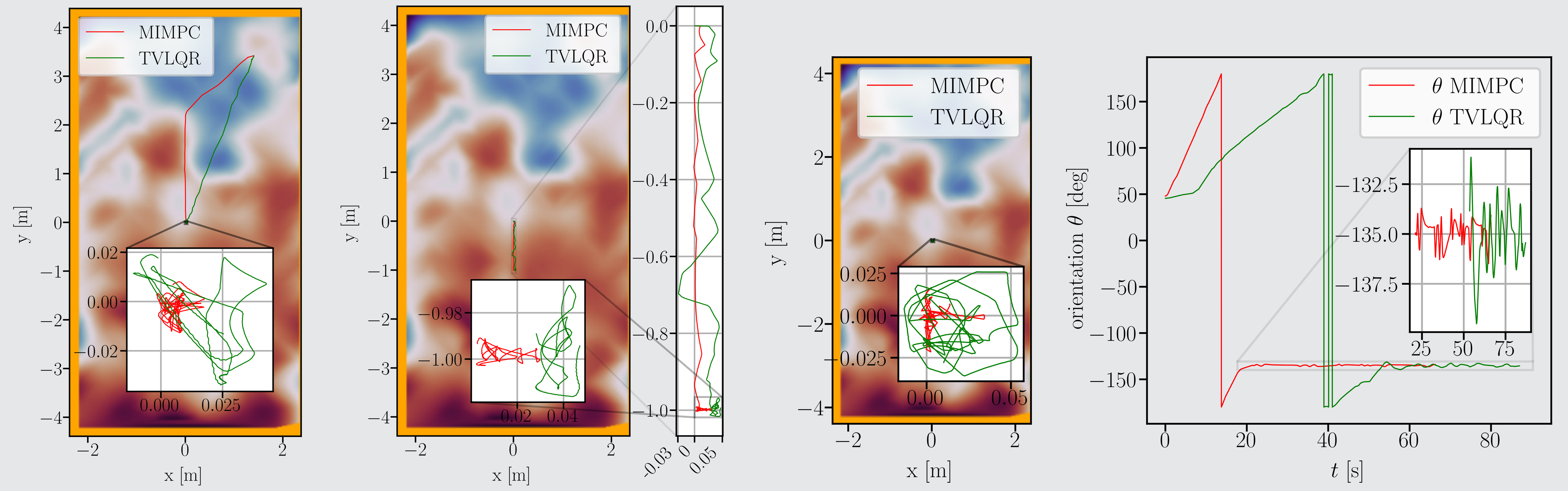
Real world experiments on REACSA

Final **MIMPC** implemented in C++ using *SCIPSolver*[3], compared to existing TVLQR

Homing	MIMPC	TVLQR
To reach limit cycle		
Time	54.00 s	87.05 s
Thrust	7.10 s	9.10 s
In limit cycle		
RMS Error	0.0086 m, 0.584°	0.0283 m, 1.140°
Oscillation	0.0089 m, 0.490°	0.030 m, 0.837°
Thrust	**0.083** $\frac{s}{s}$	0.104 $\frac{s}{s}$

1-meter	MIMPC	TVLQR
To reach limit cycle		
Time	27.00 s	47.02 s
Thrust	2.70 s	4.5 s
RMS Error	0.011 m, 0.648°	0.031 m, 1.58°
In limit cycle		
RMS Error	0.015 m, 0.739°	0.040 m, 1.78°
Oscillation	0.006 m, 0.810°	0.015 m, 0.76°
Thrust	0.089 $\frac{s}{s}$	**0.080** $\frac{s}{s}$

180 deg	MIMPC	TVLQR
To reach limit cycle		
Time	21.0 s	53.6 s
Thrust	1.90 s	4.3 s
RMS Error	0.008 m	0.027 m
In limit cycle		
RMS Error	0.013 m, 0.221°	0.019 m, 0.246°
Oscillation	0.010 m, 1.07°	0.02 m, 1.67°
Thrust	**0.050** $\frac{s}{s}$	0.060 $\frac{s}{s}$



(a) Homing (b) 1-m straight (c) rotate 180°

Figure: System trajectories on a height map of the not perfectly flat flat-floor

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